Journal of Basic and Applied Engineering Research
p-ISSN: 2350-0077; e-ISSN: 2350-0255; Volume 5, Issue 5; July-September, 2018, pp. 414-418

http://www.krishisanskriti.org/Publication.html

Tripled Point of Coincidence and Multiplicative Metric Space

Anju Panwar*[1] and Geetanjali Raiya[2]

[1]Department of Mathematics, Maharshi Dayanand University Rohtak-124001, Haryana, India
[2]Department of Mathematics, Maharshi Dayanand University Rohtak-124001, Haryana, India
*E-mail: [1]anjupanwar15@gmail.com, [2]geetanjaliraiya@gmail.com***

Abstract—*This paper represents a new tripled coincidence point using ϕ-type contractive condition for two mappings without continuity and established a new tripled point result. We also provide an example to illustrate our result.*

1. INTRODUCTION

Ozavsar and Cevikel[5] proved some fixed point theorems of multiplicative contraction mappings in multiplicative space. In 2006, Bhaskar and Lakshmikantham[4] introduced the concept of coupled fixed point and used mixed monotone property to prove some coupled fixed point theorems. Finally Borcut and Berinde [7] introduced the concept of a tripled coincidence point, which is the generalization of couple coincidence point[1].

Definition 2.1[3]. Any non-empty set M is said to be multiplicative metric space with respect to the metric $d: M\times M \rightarrow \mathbb{R}^+$ if it satisfies the following conditions:

i. $d(x, y) \geq 1$ for all $x, y \in M$,
ii. $d(x, y) = 1$ iff $x = y$ for all $x, y \in M$,
iii. $d(x, y) = d(y, x)$, for all $x, y \in M$,
iv. $d(x, y) \leq d(x, z).d(z, y)$, for all $x, y \in M$.

Definition 2.2[5]. Let M be a multiplicative metric space with respect to metric $d: M\times M \rightarrow \mathbb{R}^+$ and $\{x_n\}$ be a sequence in M and $x_0 \in X$. Then the sequence $\{x_n\}$ is said to be multiplicatively converging to x_0 written as $x_n \rightarrow x_0$, if for any multiplicative open ball

$B(x_0, \varepsilon) = \{x : d(x_0, x) < \varepsilon$ for $\varepsilon > 1\}$, there exists a natural number $N \in \mathbb{N}$ such that $n \geq N$ then $x_n \in B(x_0, \varepsilon)$. Also

$x_n \rightarrow x_0 (n\rightarrow\infty)$ iff $d(x_n, x_0) \rightarrow 1 (n \rightarrow \infty)$.

Definition 2.3[6]. Suppose (M, d) be a multiplicative metric space, then any sequence $\{x_n\}$ in X is known as multiplicative Cauchy sequence if for each $\varepsilon > 0$, there exists a positive integer $N \in \mathbb{N}$ such that $d(x_n, x_m) < \varepsilon$ for all $n, m \geq N$.

Also, $\{x_n\}$ is said to be multiplicative Cauchy sequence iff $d(x_n, x_m) \rightarrow 1$

$(n, m \rightarrow \infty)$.

Definition 2.4[6] Any multiplicative metric space (M, d) is said to be complete if every multiplicative Cauchy sequence converges in M.

Definition 2.5[8]. LetM be any non-empty set then an element $(x, y) \in M\times M$ is called coupled coincidence point of the mappings $F: M\times M \rightarrow M$ and $g: M \rightarrow M$ if $F(x, y) = gx$ and $F(y, x) = gy$

Definition 2.6[2] If M is any non-empty set then an element $(x, y, z) \in M\times M\times M$ is a tripled fixed point of the mapping $G: M\times M\times M \rightarrow M$ if

$G(x, y, z) = x$, $G(y, x, z) = y$ and $G(z, y, x) = z$.

Definition 2.7[2]. LetM be any non-empty set. An element $(x, y, z) \in M\times M\times M$is called

a) Tripled coincidence point of the mappings $G: M\times M\times M \rightarrow M$ and $h: M \rightarrow M$ if $G(x, y, z) = hx$, $G(y, x, z) = hy$ and $G(z, y, x) = hz$ and (hx, hy, hz) is called tripled point of coincidence.
b) A common tripled fixed point of mappings G and h if $G(x, y, z) = hx = x$, $G(y, x, z) = hy = y$ and $G(z, y, x) = hz = z$.

Let Φ denote the set of functions

$\phi : [1, \infty)^5 \rightarrow [0, \infty)$ satisfying

i) ϕ is non-decreasing and continuous in each coordinate variable;
ii) for $t \geq 1$,

$$\psi(t) = \max\{\phi(t,t,t,1,t),\ \phi(t,t,t,t,1),\ \phi(t,1,1,t,t),\ \phi(1,t,1,t,1),\ \phi(1,1,t,1,t)\} \le t$$

where we choose $\phi \in \Phi$.

We can obtain the result[1] from our result by using the condition of w_* - compatibility in which Y. Jiang, F. Gu[1] presented the existence of a unique common coupled fixed point in a multiplicative metric space.

2. MAIN RESULT

Theorem 3.1: Let (M, d) be a multiplicative metric space and G: M×M×M → M and h: M → M be two mappings. Suppose that there exists λ ∈ (0, 1) such that the condition

$$d(G(x, y, z), G(u, v, w))\, d(G(y, x, z), G(v, u, w))\, d(G(z, y, x), G(w, v, u))$$

$$\le \phi\begin{pmatrix} d^{\lambda}(hx,hu).d^{\lambda}(hy,hv).d^{\lambda}(hz,hw), \\ d^{\lambda}(G(x,y,z),hx).d^{\lambda}(G(y,x,z),hy).d^{\lambda}(G(z,y,x),hz), \\ d^{\lambda}(G(u,v,w),hu).d^{\lambda}(G(v,u,w),hv).d^{\lambda}(G(w,v,u),hw), \\ d^{\lambda}(G(x,y,z),hu).d^{\lambda}(G(y,x,z),hv).d^{\lambda}(G(z,y,x),hw), \\ d^{\lambda}(G(u,v,w),hx).d^{\lambda}(G(v,u,w),hy).d^{\lambda}(G(w,v,u),hv) \end{pmatrix} \quad ..(1)$$

holds for all (x, y, z), (u, v, w) ∈ M×M×M.

If G(M×M×M) ⊂ h(M) is a multiplicative complete subspace of M then G and h have a unique tripled point of coincidence, that means there exists (x, y, z) ∈M×M×M such that h(x)= G(x, y, z), h(y) = G(y, x, z)and h(z) = G(z, y, x) .

Proof: Let $x_0, y_0, z_0 \in$ M.

Since we have G(M×M×M) ⊂ h(M) then we can choose x_1, y_1, z_1 ∈M such that h x_1 = G(x_0, y_0, z_0) and h y_1 = G(y_0, x_0, z_0) and h z_1 = G(z_0, y_0, x_0).

In the same way we can choose (x_2, y_2, z_2) ∈M such that h x_2 = G(x_1, y_1, z_1) and h y_2 = G(y_1, x_1, z_1) and also h z_2 = G(z_1, y_1, x_1).

Thus we can construct three sequences $\{x_n\}$, $\{y_n\}$ and $\{z_n\}$ in M, such that

$hx_{n+1} = G(x_n, y_n, z_n)$, $hy_{n+1} = G(y_n, x_n, z_n)$ and $hz_{n+1} = G(z_n, y_n, x_n)$ for all $n \ge 0$ ..(2)

Now, from (1), (2) and by using definition of multiplicative metric space and also the function ϕ and property of ψ, we get,

$$d(hx_n, hx_{n+1})\, d(hy_n, hy_{n+1})\, d(hz_n, hz_{n+1})$$

$$= d(G(x_{n-1}, y_{n-1}, z_{n-1}), G(x_n, y_n, z_n))\, d(G(y_{n-1}, x_{n-1}, z_{n-1}), G(y_n, x_n, z_n))\, d(G(z_{n-1}, y_{n-1}, x_{n-1}), G(z_n, y_n, x_n)).$$

$$\le \phi\begin{pmatrix} d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n), \\ d^{\lambda}(G(x_{n-1},y_{n-1},z_{n-1}),hx_{n-1}).d^{\lambda}(G(y_{n-1},x_{n-1},z_{n-1}),hy_{n-1}). \\ d^{\lambda}(G(z_{n-1},y_{n-1},x_{n-1}),hz_{n-1}), \\ d^{\lambda}(G(x_n,y_n,z_n),hx_n).d^{\lambda}(G(y_n,x_n,z_n),hy_n). \\ d^{\lambda}(G(z_n,y_n,x_n),hz_n), \\ d^{\lambda}(G(x_{n-1},y_{n-1},z_{n-1}),hx_n).d^{\lambda}(G(y_{n-1},x_{n-1},z_{n-1}),hy_n). \\ d^{\lambda}(G(z_{n-1},y_{n-1},x_{n-1}),hz_n), \\ d^{\lambda}(G(x_n,y_n,z_n),hx_{n-1}).d^{\lambda}(G(y_n,x_n,z_n),hy_{n-1}). \\ d^{\lambda}(G(z_n,y_n,x_n),hz_{n-1}) \end{pmatrix}$$

$$\le \phi\begin{pmatrix} d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n), \\ d^{\lambda}(hx_n,hx_{n-1}).d^{\lambda}(hy_n,hy_{n-1}).d^{\lambda}(hz_n,hz_{n-1}), \\ d^{\lambda}(hx_{n+1},hx_n).d^{\lambda}(hy_{n+1},hy_n).d^{\lambda}(hz_{n+1},hz_n), \\ 1, d^{\lambda}(hx_{n+1},hx_n).d^{\lambda}(hx_n,hx_{n-1}), \\ d^{\lambda}(hy_{n+1},hy_n).d^{\lambda}(hy_n,hy_{n-1}), \\ d^{\lambda}(hz_{n+1},hz_n).d^{\lambda}(hz_n,hz_{n-1}) \end{pmatrix} \le$$

$$\phi\begin{pmatrix} d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n).d^{\lambda}(hx_n,hx_{n+1}) \\ d^{\lambda}(hy_n,hy_{n+1}).d^{\lambda}(hz_n,hz_{n+1}), \\ d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n).d^{\lambda}(hx_n,hx_{n+1}) \\ .d^{\lambda}(hy_n,hy_{n+1}).d^{\lambda}(hz_n,hz_{n+1}), \\ d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n).d^{\lambda}(hx_n,hx_{n+1}) \\ .d^{\lambda}(hy_n,hy_{n+1}).d^{\lambda}(hz_n,hz_{n+1}),1 \\ d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n).d^{\lambda}(hz_{n-1},hz_n).d^{\lambda}(hx_n,hx_{n+1}) \\ .d^{\lambda}(hy_n,hy_{n+1}).d^{\lambda}(hz_n,hz_{n+1}), \end{pmatrix}$$

$$\le \psi\begin{pmatrix} d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n). \\ d^{\lambda}(hz_{n-1},hz_n) \\ .d^{\lambda}(hx_n,hx_{n+1}).d^{\lambda}(hy_n,hy_{n+1}). \\ d^{\lambda}(hz_n,hz_{n+1}) \end{pmatrix}$$

$$\le \begin{pmatrix} d^{\lambda}(hx_{n-1},hx_n).d^{\lambda}(hy_{n-1},hy_n). \\ d^{\lambda}(hz_{n-1},hz_n) \\ .d^{\lambda}(hx_n,hx_{n+1}).d^{\lambda}(hy_n,hy_{n+1}). \\ d^{\lambda}(hz_n,hz_{n+1}) \end{pmatrix}$$

$$d^{1-\lambda}(hx_n, hx_{n+1})\, d^{1-\lambda}(hy_n, hy_{n+1})\, d^{1-\lambda}(hz_n, hz_{n+1}) \le d^{\lambda}(hx_{n-1}, hx_n)\, d^{\lambda}(hy_{n-1}, hy_n).d^{\lambda}(hz_{n-1}, hz_n)$$

Taking power $\frac{1}{1-\lambda}$ both sides, we get

$d(hx_n, hx_{n+1}).\ d(hy_n, hy_{n+1}).d(hz_n, hz_{n+1})$

$\leq d^{\frac{\lambda}{1-\lambda}}(hx_{n-1}, hx_n) d^{\frac{\lambda}{1-\lambda}}(hy_{n-1}, hy_n) d^{\frac{\lambda}{1-\lambda}}(hz_{n-1}, hz_n)$

$= d^t(hx_{n-1}, hx_n)\ d^t(hy_{n-1}, hy_n).\ d^t(hz_{n-1}, hz_n)$ .. (3)

Where $t = \frac{\lambda}{1-\lambda} \in (0, 1)$.

From equation (3), we get

$d\ (hx_n, hx_{n+1}).\ d\ (hy_n, hy_{n+1}).d(hz_n, hz_{n+1})$

$\leq d^t(hx_{n-1}, hx_n)\ d^t(hy_{n-1}, hy_n).\ d^t(hz_{n-1}, hz_n)$

.

.

$\leq d^{t^n}(hx_0, hx_1).d^{t^n}(hy_0, hy_1).d^{t^n}(hz_0, hz_1)$

Thus for all n ∈ℕ, n < m, by multiplicative triangular inequality, we obtain

$d(hx_n, hx_m).d(hy_n, hy_m)\ d(hz_n, hz_m)$

$\leq d(hx_n, hx_{n+1}).d(hx_{n+1}, hx_{n+2})\ldots d(hx_{m-1}, hx_m)\ d(hy_n, hy_{n+1}).d(hy_{n+1}, hy_{n+2})\ldots d(hy_{m-1}, hy_m)\ d(hz_n, hz_{n+1}).d(hz_{n+1}, hz_{n+2})\ldots d(hz_{m-1}, hz_m)$

$\leq d^{t^n}(hx_0, hx_1).d^{t^n}(hy_0, hy_1).d^{t^n}(hz_0, hz_1)$

$d^{t^{n+1}}(hx_0, hx_1).d^{t^{n+1}}(hy_0, hy_1).d^{t^{n+1}}(hz_0, hz_1)$

…

$d^{\frac{t^n}{1-t}}(hx_0, hx_1).d^{\frac{t^n}{1-t}}(hy_0, hy_1).d^{\frac{t^n}{1-t}}(hz_0, hz_1)$

Implies that

$d(hx_n, hx_m)\ d(hy_n, hy_m)\ d(hz_n, hz_m) \to 1\ (n, m \to \infty)$ such that

$d(hx_n, hx_m) \to 1\ (n, m \to \infty)$

$d(hy_n, hy_m) \to 1\ (n, m \to \infty)$

$d(hz_n, hz_m) \to 1\ (n, m \to \infty)$

Hence $\{hx_n\}$, $\{hy_n\}$ and $\{hz_n\}$ are Cauchy sequence in h(X).

Now, by completeness of h(X), there exist hx, hy, hz∈ h(X) such that $\{hx_n\}$, $\{hy_n\}$ and $\{hz_n\}$ converges to hx, hy and hz respectively.

Now we claim that G(x, y, z) = hx, G(y, x, z) = hy and G(z, y, x) = hz.

It follows from multiplicative triangle inequality and equation (1) that

$d(G(x, y, z), hx)\ d(G(y, x, z), hy)\ d(G(z, y, x), hz)$

$\leq d(G(x, y, z), hx_{n+1}).d(hx_{n+1}, hx).\ d(G(y, x, z), hy_{n+1})d(hy_{n+1}, hy)\ d(G(z, y, x), hz_{n+1})\ d(hz_{n+1}, hz)$

$= d(G(x, y, z), G(x_n, y_n, z_n))\ d(G(y, x, z), G(y_n, x_n, z_n))\ d(G(z, y, x), G(z_n, y_n, x_n))$

$$\leq \phi\begin{pmatrix} d^\lambda(hx, hx_n).d^\lambda(hy, hy_n).d^\lambda(hz, hz_n), \\ d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy).d^\lambda(G(z,y,x), hz), \\ d^\lambda(G(x_n,y_n,z_n), hx_n).d^\lambda(G(y_n,x_n,z_n), hy_n).d^\lambda(G(z_n,y_n,x_n), hz_n), \\ d^\lambda(G(x,y,z), hx_n).d^\lambda(G(y,x,z), hy_n).d^\lambda(G(z,y,x), hz_n), \\ d^\lambda(G(x_n,y_n,z_n), hx).d^\lambda(G(y_n,x_n,z_n), hy).d^\lambda(G(z_n,y_n,x_n), hz). \end{pmatrix}$$

$$.\ d^\lambda(hx_{n+1}, hx).d^\lambda(hy_{n+1}, hy)d^\lambda(hz_{n+1}, hz)$$

$$\leq \phi\begin{pmatrix} d^\lambda(hx, hx_n).d^\lambda(hy, hy_n).d^\lambda(hz, hz_n), \\ d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy).d^\lambda(G(z,y,x), hz), \\ d^\lambda(hx_{n+1}, hx_n).d^\lambda(hy_{n+1}, hy_n).d^\lambda(hz_{n+1}, hz_n), \\ d^\lambda(G(x,y,z), hx_n).d^\lambda(G(y,x,z), hy_n).d^\lambda(G(z,y,x), hz_n), \\ d^\lambda(hx_{n+1}, hx).d^\lambda(hy_{n+1}, hy).d^\lambda(hz_{n+1}, hz). \end{pmatrix}.$$

$$d^\lambda(hx_{n+1}, hx).d^\lambda(hy_{n+1}, hy)d^\lambda(hz_{n+1}, hz)$$

Applying $n \to \infty$ in above inequality, we get

$d(G(x, y, z), hx)d(G(y, x, z), hy)\ d(G(z, y, x), hz)$

$$\leq \phi\begin{pmatrix} 1, d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy). \\ d^\lambda(G(z,y,x), hz), 1, \\ d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy). \\ d^\lambda(G(z,y,x), hz), 1 \end{pmatrix} \quad 1.1.1$$

$$\leq \psi\left(d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy).d^\lambda(G(z,y,x), hz)\right)$$

$$\leq \left(d^\lambda(G(x,y,z), hx).d^\lambda(G(y,x,z), hy).d^\lambda(G(z,y,x), hz)\right)$$

Since we have λ ∈ (0, 1) then it implies

d(G(x, y, z), hx).d(G(y, x, z), hy).d(G(z, y, x), hz)= 1

Implies that

d(G(x, y, z), hx) = 1, d(G(y, x, z), hy) = 1, d(G(z, y, x), hz) = 1.

Then G(x, y, z) = hx, G(y, x, z) = hy, G(z, y, x) = hz.

Hence, (hx, hy, hz) is a tripled point of coincidence of mappings G and h.

Now, we claim that this point of coincidence is unique.

For this, let us suppose that there is another (x*, y*, z*) ∈ M×M×M such that (hx*, hy*, hz*) is a tripled point of mapping G and h,

Then, by equation (1), we have

d(hx, hx*).d(hy, hy*).d(hz, hz*)

= d(G(x, y, z),G(x*, y*, z*)) . d(G(y, x, z),(G(y*, x*, z*)). d(G(z, y, x),G(z*, y*, x*))

$$\leq \phi\begin{pmatrix} d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*), \\ d^{\lambda}(hx,hx).d^{\lambda}(hy,hy).d^{\lambda}(hz,hz), \\ d^{\lambda}(hx^*,hx^*).d^{\lambda}(hy^*,hy^*).d^{\lambda}(hz^*,hz^*), \\ d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*), \\ d^{\lambda}(hx^*,hx).d(hy^*,hy).d^{\lambda}(hz^*,hz) \end{pmatrix}$$

$$= \phi\begin{pmatrix} d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*), \\ 1,1, \\ d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*), \\ d^{\lambda}(hx^*,hx).d(hy^*,hy).d^{\lambda}(hz^*,hz) \end{pmatrix}$$

$$\leq \psi\left(d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*)\right)$$

$$\leq \left(d^{\lambda}(hx,hx^*).d^{\lambda}(hy,hy^*).d^{\lambda}(hz,hz^*)\right)$$

From this inequality, we have

$$\left(d(hx,hx^*).d(hy,hy^*).d(hz,hz^*)\right) = 1$$

Implies

d(hx, hx*) = 1, d(hy, hy*) = 1, d(hz, hz*) = 1

We get,

hx = hx*, hy = hy*and hz = hz*

Hence (hx, hy, hz) is a unique tripled point of coincidence of mappings G and h.

Example 2.8. Let (X, d) be a multiplicative metric space where X = [0, 2] and d(x, y) = $e^{|x-y|}$ for all x, y ∈ M. Let G: M×M×M → M and h: M →M be two mappings defined by

G(x, y, z) = x andhx = 10x for all x, y ∈ M.

We have

d(G(x, y, z),G(u, v, w))d(G(y, x, z),G(v, u, w)) d(G(z, y, x),G(w, v, u)) = d(x, u).d(y, v).d(z, w)

$$= e^{|x-u|}e^{|y-v|}e^{|z-w|}$$

$$= e^{|x-u|+|y-v|+|z-w|}$$

$$= e^{|(x+y+z)-(u+v+w)|}$$

From the condition (1) with $\lambda = \frac{1}{2}$ and $\phi(t_1,t_2,t_3,t_4,t_5) = t_1.t_2.t_3.t_4.t_5$ we have

$$\phi\begin{pmatrix} d^{\lambda}(hx,hu).d^{\lambda}(hy,hv).d^{\lambda}(hz,hw), \\ d^{\lambda}(G(x,y,z),hx).d^{\lambda}(G(y,x,z),hy).d^{\lambda}(G(z,y,x),hz), \\ d^{\lambda}(G(u,v,w),hu).d^{\lambda}(G(v,u,w),hv).d^{\lambda}(G(w,v,u),hw), \\ d^{\lambda}(G(x,y,z),hu).d^{\lambda}(G(y,x,z),hv).d^{\lambda}(G(z,y,z),hw), \\ d^{\lambda}(G(u,v,w),hx).d^{\lambda}(G(v,u,w),hy).d^{\lambda}(G(w,v,u),hv) \end{pmatrix}$$

$$= \phi\begin{pmatrix} d^{\lambda}(10x,10u).d^{\lambda}(10y,10v).d^{\lambda}(10z,10w), \\ d^{\lambda}(x,10x).d^{\lambda}(y,10y).d^{\lambda}(z,10z), \\ d^{\lambda}(u,10u).d^{\lambda}(v,10v).d^{\lambda}(w,10w), \\ d^{\lambda}(x,10u).d^{\lambda}(y,10v).d^{\lambda}(z,10w), \\ d^{\lambda}(u,10x).d^{\lambda}(v,10y).d^{\lambda}(w,10z) \end{pmatrix}$$

$$= \phi\begin{pmatrix} e^{|10x-10u|\cdot\frac{1}{2}}.e^{|10y-10v|\cdot\frac{1}{2}}.e^{|10z-10w|\cdot\frac{1}{2}}, e^{|x-10x|\cdot\frac{1}{2}}.e^{|y-10y|\cdot\frac{1}{2}}.e^{|z-10z|\cdot\frac{1}{2}}, \\ e^{|u-10u|\cdot\frac{1}{2}}.e^{|v-10v|\cdot\frac{1}{2}}.e^{|w-10w|\cdot\frac{1}{2}}, e^{|x-10u|\cdot\frac{1}{2}}.e^{|y-10v|\cdot\frac{1}{2}}.e^{|z-10w|\cdot\frac{1}{2}}, \\ e^{|u-10x|\cdot\frac{1}{2}}.e^{|v-10y|\cdot\frac{1}{2}}.e^{|w-10z|\cdot\frac{1}{2}} \end{pmatrix}$$

$$= \begin{pmatrix} e^{|10x-10u|\cdot\frac{1}{2}}.e^{|10y-10v|\cdot\frac{1}{2}}.e^{|10z-10w|\cdot\frac{1}{2}}.e^{|x-10x|\cdot\frac{1}{2}}.e^{|y-10y|\cdot\frac{1}{2}}.e^{|z-10z|\cdot\frac{1}{2}}. \\ e^{|u-10u|\cdot\frac{1}{2}}.e^{|v-10v|\cdot\frac{1}{2}}.e^{|w-10w|\cdot\frac{1}{2}}.e^{|x-10u|\cdot\frac{1}{2}}.e^{|y-10v|\cdot\frac{1}{2}}.e^{|z-10w|\cdot\frac{1}{2}}. \\ e^{|u-10x|\cdot\frac{1}{2}}.e^{|v-10y|\cdot\frac{1}{2}}.e^{|w-10z|\cdot\frac{1}{2}} \end{pmatrix}$$

$$\geq e^{\left|-10(u+v+w)-5(x+y+z)+\frac{1}{2}(x+y+z)+\frac{3}{2}(u+v+w)\right|}$$

$$\geq e^{|(x+y+z)-(u+v+w)|}$$

≥ d(G(x, y, z),G(u, v, w))d(G(y, x, z), G(v, u, w))d(G(z, y, x),G(w, v, u))

For all x, y, z ∈ M×M×M.

Hence all the conditions of theorem 2.1 are satisfied. Moreover (0, 0, 0) is the tripled coincidence point of mappings G and h.

REFERENCES

[1] Y. Jiang and F. Gu; Common coupled fixed point results in multiplicative metric space and applications, J. Nonlinear Sci. Appl., 10 (2017), 1881−1895.

[2] HassenAydi, Erdal Karapinar and Mihai Postolache; Tripled coincidence point theorems for weak φ-contractions in partially ordered metric spaces,Aydi et al; licensee Springer. 2012, 1-12.

[3] Y. J. Cho, M. H. Shah and N. Hussain; Coupled fixed points of weakly F-contractive mappings in topological spaces, Appl. Math. Lett., 24 (2011), 1185-1190.

[4] T. GnanaBhaskar and V Lakshmikantham; Fixed point theorems in partially ordered metric spaces and applications, Non- linear Anal., 65 (2006), 1379-1393.
[5] M. Özavsar and A. C. Cevikel; Fixed points of multiplicative contraction mappings on multiplicative metric spaces, ArXiv, 2012 (2012), 14 pages.
[6] W. Shatanawi; Onw-compatible mappings and common coupled coincidence point in cone metric spaces, Appl. Math. Lett., 25 (2012), 925-931.
[7] M. Borcut and V. Berinde; Tripled coincidence theorems for contractive type mappings in partially ordered metric spaces, Appl. Math. Comput. 218(2012), 5929-5936.
[8] V. Lakshmikantham and Lj. C´iric´; Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces, Nonlinear Anal., **70** (2009), 4341–4349.